AT622 Section 3
Basic Laws

There are three stages in the life of a photon that interest us: first it is created, then it propagates through space, and finally it can be destroyed. The creation and destruction of a photon occurs through its interaction with matter. Here we consider the basic laws that characterize the creation of radiation by a process referred to as emission. Processes that destroy the photon, via absorption, are topics of later chapters.

3.1 Equilibrium Radiation and Kirchoff's Law

The generation of electromagnetic waves occurs as a general result of an accelerating electric charge. In general, any object is composed of a vast number of molecules that oscillate over a continuous range of frequencies and therefore emit radiation of all frequencies. However, this radiation is not emitted equally at all frequencies but is distributed in some way according to the emission spectrum, which, as we shall see, depends strongly on the temperature of the object.

The nature of the emission spectrum and its relationship to the temperature of the body loomed as a major challenge to physicists in the late nineteenth century. In fact, the relationship could not be accounted for using the principles of classical physics and its description marked one of the major turning points in the history of science. In attempting to formulate the description of the emission spectrum there emerged the hypothetical concept of a blackbody, which is a body whose surface absorbs all radiation incident upon it. It also follows that any two blackbodies at the same temperature emit precisely the same radiation and that a blackbody emits more radiation than any other type of object at the same temperature.

That it is more appropriate to view blackbody radiation as equilibrium radiation is evident by considering an isolated cavity with walls opaque to all radiation. The cavity walls constantly emit, absorb, and reflect radiation until a state of equilibrium is reached (i.e., until the temperature of the cavity walls no longer change in time). This equilibrium radiation fills the cavity uniformly and is just the same as the radiation emitted by a hypothetical blackbody at the same temperature of the cavity. To understand why this is so, imagine that a blackbody is placed in the cavity. This body absorbs the entire equilibrium radiation incident on its surface and, since the cavity is in a state of equilibrium, the radiation emitted by the object must be precisely that absorbed by it, which also happens to be the equilibrium radiation that fills the cavity. Therefore under the conditions of equilibrium, the ability of a body to radiate is closely related to its ability to absorb radiation. The mathematical formulation of this statement is known as Kirchoff's Law, which can be written as

$$E_\lambda(T) = \varepsilon_\lambda B_\lambda(T) \tag{3.1}$$

where E_λ *is* the emitted radiation and $B_\lambda(T)$ is the radiation of the hypothetical blackbody. E_λ *is* sometimes referred to as the spectral emissive power and the total emissive power is

$$E(T) = \int_0^\infty E_\lambda(T)\,d\lambda$$

The proportionality constant in Eqn. (3.1) is referred to as an emissivity, ε (sometimes also referred to as an absorption coefficient), which in this context varies between 0 and 1. If $\varepsilon_\lambda = 0$, then Eqn. (3.1) states that a body neither emits radiation at the given wavelength nor absorbs radiation at the same wavelength.

For $\varepsilon_\lambda = 1$ on the other hand, the emitted radiation is just blackbody radiation and the body absorbs all radiation incident upon it. As we shall see in following sections, the absorption coefficient contains information about the type of matter that emits radiation. The wavelength dependence of this coefficient varies dramatically according to the nature of the matter and the portion of the electromagnetic spectrum under consideration.

Table 3.1 Typical gray body emissivities and reflectivities for various 'opaque' surfaces. These quantities are averaged over respective terrestrial and solar emission spectra (later sections). Albedo (α) refers to the reflectivity of solar radiation. Because the sun and earth are not in thermal equilibrium, blackbody relationships between emission, absorption and reflection do not apply.

Type	Emissivity (ε)	Albedo (α)
Tropical forest	0.13	0.99
Woodland	0.14	0.98
Farmland/natural grassland	0.20	0.95
Semi-desert/stony desert	0.24	0.92
Dry sandy desert/salt pans	0.37	0.89
Water (0°-60°)[a]	<0.08	0.96
Water (60°-90°)[a]	<0.10	0.96
Sea ice	0.25-0.60	0.90
Snow-covered vegetation	0.20-0.80	0.88
Snow-covered ice	0.80	0.92

[a]The albedo of a water surface increases as the solar zenith angle increases. Ocean surface albedos are also increased by the occurrence of white caps on the waves.

Gray bodies: ε_λ is assumed constant and independent of λ.

It is through the statement of Kirchoff's Law that the whole point of blackbody radiation is relevant. All black bodies at some temperature behave identically and the radiation emitted by such bodies at a given λ depends only on the temperature of the body. Thus the emission of radiation at some chosen wavelength is solely determined by the characteristics of the emitting matter (through a_λ) and temperature (through B_λ).

Example 3.1: Show that two blackbodies at the same temperature must emit the same radiation.

Proof of this lies in the second law of thermodynamics. In the case of two black surfaces *A* and *B* at the same temperature, suppose *A* radiates more energy than the other. Imagine placing these surfaces next to each other and allowing each to absorb the radiation from the other. Thus *B* must absorb more radiation than it emits, receiving more energy and becoming hotter. *A*, correspondingly becomes cooler. Thus the second law of thermodynamics is violated and our assumption that *A* radiates more than *B* is false.

3.2 Planck's Black Body Function and Related Laws

The theoretical question of what form the wavelength distribution of the intensity of this cavity radiation takes and how this radiation in turn depends on the temperature of the walls of the cavity occupied the attention of many of the worlds leading physicists during the 1890's. It was Max Planck who provided us with the theoretical description of the black body radiation but in doing so he was forced to make an assumption that proved to be one of the most daring departures from the philosophies of physics at that time. He considered that each of the oscillators in the walls of the cavity could have only one of a discrete set of energies rather than the more conventional view that energy could assume any value above or equal to zero. The discrete energy level of the oscillator could then be represented in the form

$$E = nhv$$

where n is an integer, referred to as the quantum number, which defines the permitted number of discrete units of energy of the oscillator. The fundamental unit of energy turned out to be proportional to the frequency of the oscillator v where the proportionality constant h is known as Planck's constant. It is these discrete packets of quanta of energy that are emitted by the oscillators in the cavity walls after the oscillator undergoes a transition from one quantized energy state to another. On the basis of these arguments, Planck was able to demonstrate that the relationship,

$$B_\lambda = (T) = \frac{2hc^2}{\lambda^5[e^{hc/K\lambda T} - 1]}, \qquad (3.2a)$$

adequately describes black body radiation where K is Boltzman's constant and T is the absolute temperature of the cavity walls. It is also customary to introduce the constants

$$C_1 = 2\pi hc_o^2 = 3.7419 \times 10^{-16} \qquad \mathrm{Wm}^{-2}$$

in which case

$$B_\lambda(T) = \frac{C_1}{\pi\lambda^5[e^{C_2/\lambda T} - 1]}, \qquad (3.2b)$$

where it is assumed for convenience that $c = c_o$.

The function defined by Eqn. (3.2) is known as Planck's function and is graphically portrayed in Fig. 3.1 for six different temperatures. These examples demonstrate a gross relationship that perhaps could have been anticipated. For example, consider an ordinary electrical element on a stove. On the highest and thus hottest setting the element glows brightest with a reddish hue (Fig. 3.1). When the electricity is turned off and the element is allowed to cool, the color of the element fades until its luminosity vanishes. But it is still radiating; a fact evident when a hand is placed above the cooling element. This simple experiment known as Wein's displacement law, which establishes a connection between the wavelength of maximum emission (λ_{max}) and the temperature of the radiator. This law is simply derived from

$$\frac{\partial B_\lambda}{\partial \lambda} = 0$$

from which it follows that

$$T\lambda_{max} = 2898 \ (\mu\text{m} \ ^\circ\text{K}). \tag{3.3}$$

Wein's displacement law is also graphically depicted on Fig. 3.1 as the diagonal line joining the maxima of the three Planck functions.

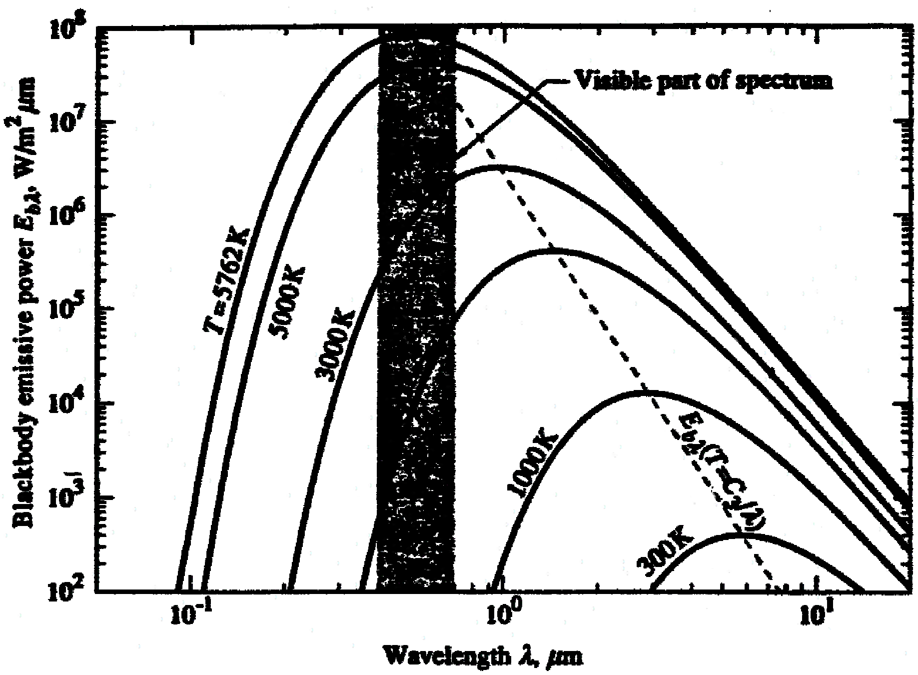

Fig. 3.1 Planck's black body flux curve at the three temperatures shown. The units of this function are Wm^{-2} μm^{-1}. The diagonal line intersecting the curves at their maxima depicts Wein's displacement law.

Example 3.2: What is the wavelength of the maximum emissive power of the sun? What is the corresponding wavelength of Earth? The temperature of the sun is approximately 5760K and it follows from Eqn. (3.3) that

$$\lambda_{\max} = \frac{2898}{5760} = 0.5 \ \mu\text{m}$$

which roughly corresponds to the middle of the visible portion of the spectrum (Fig. 3.2a). Solar radiation is attenuated as it penetrates the atmosphere. Understanding this attenuation in some detail is one of the goals of this course.

Temperatures of emitters in the Earth's atmosphere vary. Assuming a value of 290K, it follows that

$$\lambda_{\max} = \frac{2898}{290} = 10 \quad \mu\text{m}$$

Example 3.2 Continued.

Figure 3.2b is an example of the emission spectrum at the top of the atmosphere measured at one location. This spectral emission does not follow the black body curve since it occurs through a kind of transfer from layer to layer in the atmosphere through a combination of absorption at low levels and emission at higher levels and at colder temperatures. The difference between the measured emission and that of a black body is crudely indicative of the absorption spectrum of the absorbing gases in the atmosphere. The transfer of radiation and a detailed understanding of the absorption spectrum are topics that we will return to later.

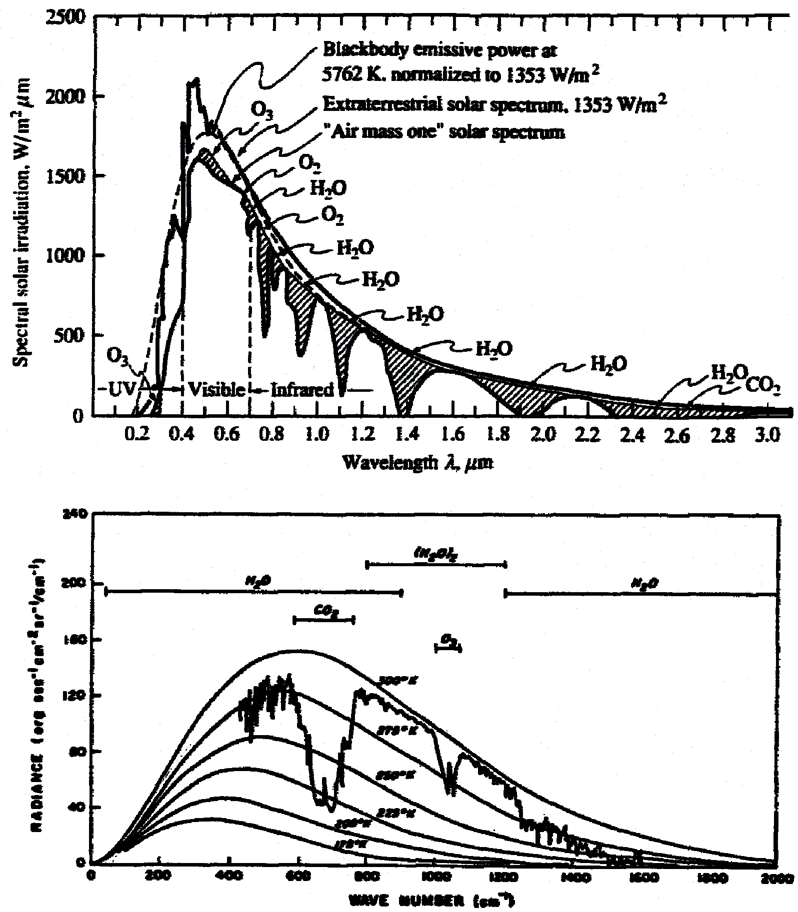

Fig. 3.2 *(a) Solar irradiation measured at the top of the Earth's atmosphere compared to that of a 5760K black body normalized to 1353 Wm^{-2} (the reason for this will be discussed later). Also shown in a schematic way is the irradiance at the surface under 'typical' clear sky conditions. (b) The spectrum of infrared radiation emitted to space from Earth as measured by an instrument on an orbiting satellite. This spectrum corresponds to clear sky conditions over the Saharan desert.*

There follows from Eqn. (3.2) two important limits of the Planck function. The first of these limits is referred to as Wein's distribution and applies to $\lambda \rightarrow 0$

$$B_{\tilde{\nu}} = 2h\tilde{\nu}c^2 e^{-hc\tilde{\nu}/KT} \tag{3.4}$$

whereas the longer wavelength limit, $\lambda \rightarrow \infty$ is referred to as the Rayleigh-Jeans distribution, and is expressed by

$$B_{\tilde{\nu}} = 2cKT\tilde{\nu}^2 \tag{3.5}$$

This long wave limit has a direct application to passive microwave remote sensing problems. At these wavelengths, the emission by the earth's atmosphere is directly proportional to temperature and intensity and temperature can be treated as mutually equivalent. We refer to the intensity expressed in units of temperature as the brightness temperature, which is the temperature that is required to match that measured intensity to the Planck black body function. For microwave radiation, this is simply obtained from Eqn. (3.5). At other wavelengths, the brightness temperature is obtained by inverting either Eqn. (3.2) or Eqn. (3.4).

3.3 Total Blackbody Emissive Power

An obvious characteristic of blackbody radiation is that the hotter the object, the greater the total amount of radiation is emitted from a given surface area. This is just a statement of Stefan-Boltzmann's law, which can be simply derived by integrating B_λ over the entire wavelength domain according to

$$B(T) = \int_0^\infty B_\lambda(T)d\lambda = \frac{C_1 T^4}{\pi}\int_0^\infty \frac{d(\lambda T)}{(\lambda T^5)\left[\exp^{C_2/\lambda T} - 1\right]} = \left[\frac{C_1}{\pi C_2^4}\int_0^\infty \frac{y^3 dy}{e^y - 1}\right]T^4$$

where $y = C_2,/\lambda T$. The integral in this expression is $\pi^4/15$ and the constant

$$\sigma = \frac{\pi^4 C_1}{15 C_2^4} = 5.67\times 10^{-8} \qquad \mathrm{Wm^{-2}K^{-4}}$$

is the Stefan-Boltzmann constant. The total blackbody emission (intensity) thus follows

$$B(T) = \frac{\sigma}{\pi}T^4 \tag{3.6a}$$

where the reason for the appearance of the π factor arises from the properties of isotropic radiation. The hemispheric blackbody flux is thus

$$\pi B(T) = \sigma T^4 \tag{3.6b}$$

As an example, the radiation emitted from a 6000°K blackbody, for instance, is 160,000 times that emitted from a 300°K blackbody.

It is often convenient to use the Planck function defined in terms of wavenumber rather than wavelength. The relationship between these two forms is obtained from the simple requirements that the integrated energies must be an expression of Stefan-Boltzman's law. Thus

$$B_\lambda(T)d\lambda = -B_{\tilde{v}}(T)d\tilde{v}$$

and, with Eqn. (3.2a) together with the definition of $\tilde{v}$, it follows that

$$B_{\tilde{v}}(T) = \frac{2hc^2\tilde{v}^3}{e^{ch\tilde{v}/KT} - 1} \tag{3.7}$$

Many problems in atmospheric radiation require the Planck function integrated over some finitely wide spectral region, say between λ_1, and λ_2. Then

$$\int_{\lambda_1}^{\lambda_2} B_\lambda(T)d\lambda = \left[\frac{C_1}{\pi C_2^4}\int_{y_2}^{y_1}\frac{y^3dy}{e^y-1}\right]T^4$$

which cannot be evaluated analytically. The fraction of blackbody radiation between 0 and λ_1, namely

$$f(\lambda_1 T) = \frac{\int_0^{\lambda_1} B_\lambda(T)d\lambda}{\int_0^{\infty} B_\lambda(T)d\lambda} = \frac{15}{\pi^4}\int_{y_1}^{\infty}\frac{y^3dy}{e^y-1}$$

can be evaluated numerically or using precomputed look-up tables. The spectrally integrated blackbody radiation then becomes

$$\int_{\lambda_1}^{\lambda_2} B_\lambda(T)d\lambda = \left[f(\lambda_2 T) - f(\lambda_1 T)\right]\frac{\sigma}{\pi}T^4 \tag{3.8}$$

and a program that calculates the factor in parentheses is supplied in Appendix 3A.

Example 3.3: What fraction of the total solar emission occurs at wavelengths longer than 0.7 μm?

Making use of the above mentioned program

```
program test
W2=4
W1=0.7
T=5800
frac=PLANCK(W1,W2,T)
write(*,*) 'fraction=' , frac
end
```

Distribution of the solar constant in various wavelength bands.

Band	Wavelength Interval (nm)	Irradiance ($W\ m^{-2}$)	Fraction of E_s (percent)[a]
Ultraviolet and beyond	< 350	62	4.5
Near ultraviolet	350-400	57	4.2
Visible	400-700	522	38.2
Near infrared	700-1000	309	22.6
Infrared and beyond	> 1000	417	30.5
Total		1367	100.0

[a]Percentages computed from data in Thekaekara (1976).

3.4 Problems

Problem 3.1

Assuming that the normal body temperature is 37 °C, what would the emittance (i.e., how much radiation is emitted) by the body if:

(a) The body was a perfect blackbody?
(b) The body was gray with 90% absorption?

What is the wavelength of maximum emission?

Problem 3.2

Consider a room with a fireplace, which has an opening of 1 m^2. The opening is composed of 10% flame, 30% logs and 60% walls. The flames have an emittance of 0.5, while the walls and logs are black. Assume the respective temperatures of the flames to be 2000 °K, of the logs, 1000 °K, and of the walls, 500 °K, and that only radiation energy escapes into the room. What is the total radiant power escaping the fireplace from each source and the wavelength of maximum emission from each? Explain the effect of placing a glass plate over the opening if the glass has the property of

$$\text{transmittance} = 1,\ \text{absorptance} = 0;\ 0 \le \lambda \le 3\mu$$

$$\text{transmittance} = 0,\ \text{absorptance} = 1;\ \lambda > 3\mu$$

Problem 3.3

There are two approximate forms of Planck's Law. The first is known as Wien's Law.

$$B_\lambda = c_1 \lambda^{-5} e^{-(c_2/\lambda T)} \ .$$

This expression is valid for very small values of λT. What would be the numerical value of T for which less than a 1% error would be incurred at 1 μm using the above approximation?

A second simiplification is the Rayleigh-Jeans approximation often applied to microwave wavelengths.

$$B_\lambda = c_2 \lambda^{-4} T$$

Derive the above expression from Planck's Law and define the 1% error threshold in terms of T and λ = 500 μm.

Problem 3.4

Convert the wavelength form of Planck's Law to the wavenumber form given below:

$$\mathrm{B}_v = \frac{c_1 \tilde{v}^3}{e^{c_2 \tilde{v}/T} - 1} \qquad \text{where: } \tilde{v} = \frac{1}{\lambda}; \quad d\tilde{v} = -\frac{1}{\lambda^2} d\lambda$$

Problem 3.5

Show that the maximum intensity of the Planck's function is proportional to the fifth power of the temperature. Comment.

APPENDIX 3A

```
FUNCTION PLANCK(Wl,W2,T)
C
C   Use an approximate integral scheme to evaluate the integral
C   of Planck's Law. W1 and W2 define the upper and lower wavelength extent of
C   the band in micron and T is temperature in K.
C   Output is in units of W sq m per ster (radiance units)
C   Ref:
C
       WVN1=1./(Wl*I.E-4)
       WVN2=1./(W2*1.E-4)
       X1=1.43868*WVN1/T
       X2=1.43868*WVN2/T
C
C   CALCULATE THE MEAN PLANCK FUNCTION
C
C
       PLANCK=ABS(PL(X1)-PL(X2))*SIGMAP*T**4
       write(*,*) X1,X2,WVN1,WVN2,T
       PLANCK=ABS(PL(X1)-PL(X2))
C
       RETURN
       END
 ------------------------------------------------------------------------
C
FUNCTION PL(X)
C
       INTEGER MM
       PI=3.1415926
       PL=0.0
       IF (X.GT.2.5) THEN
       DO 101 MM=1,50
       M=FLOAT(MM)
       TERM=EXP(-M*X)*(((M*X+3.)*M*X+6.)*M*X+6.)*15.0/(PI*M)**4
       PL=PL+TERM
       IF (ABS(TERM/PL).LT.1.0E-5) RETURN
101    CONTINUE
       ELSE
       PL=1.0-15.0/PI**4*x**3*
      $(1./3.-X/8.+X**2/60.-X**4/5040.+X**6/272160.-X**8/13305600.)
       ENDIF
       RETURN
       END
```