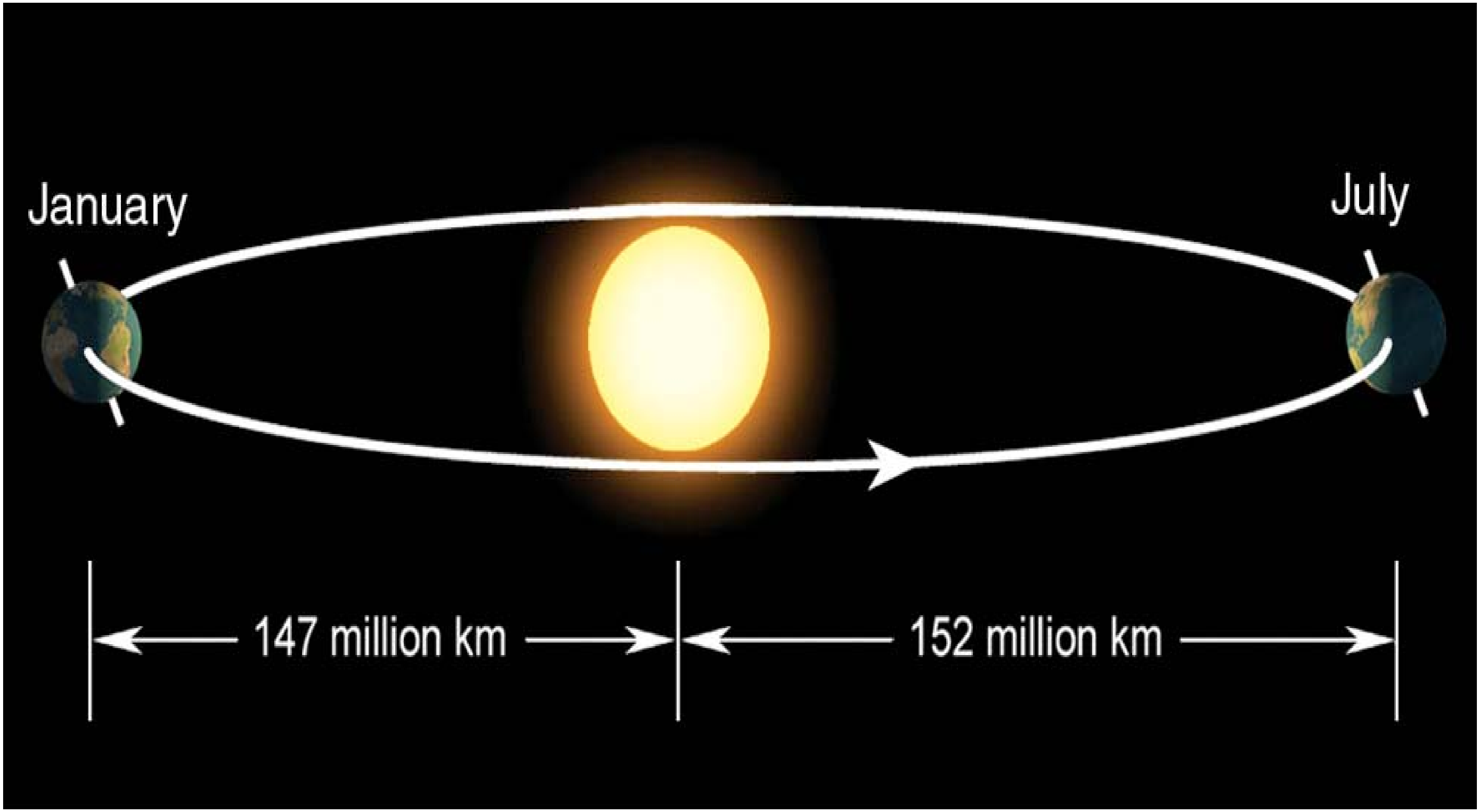
AOSC400-2015
September 17, Lecture # 5

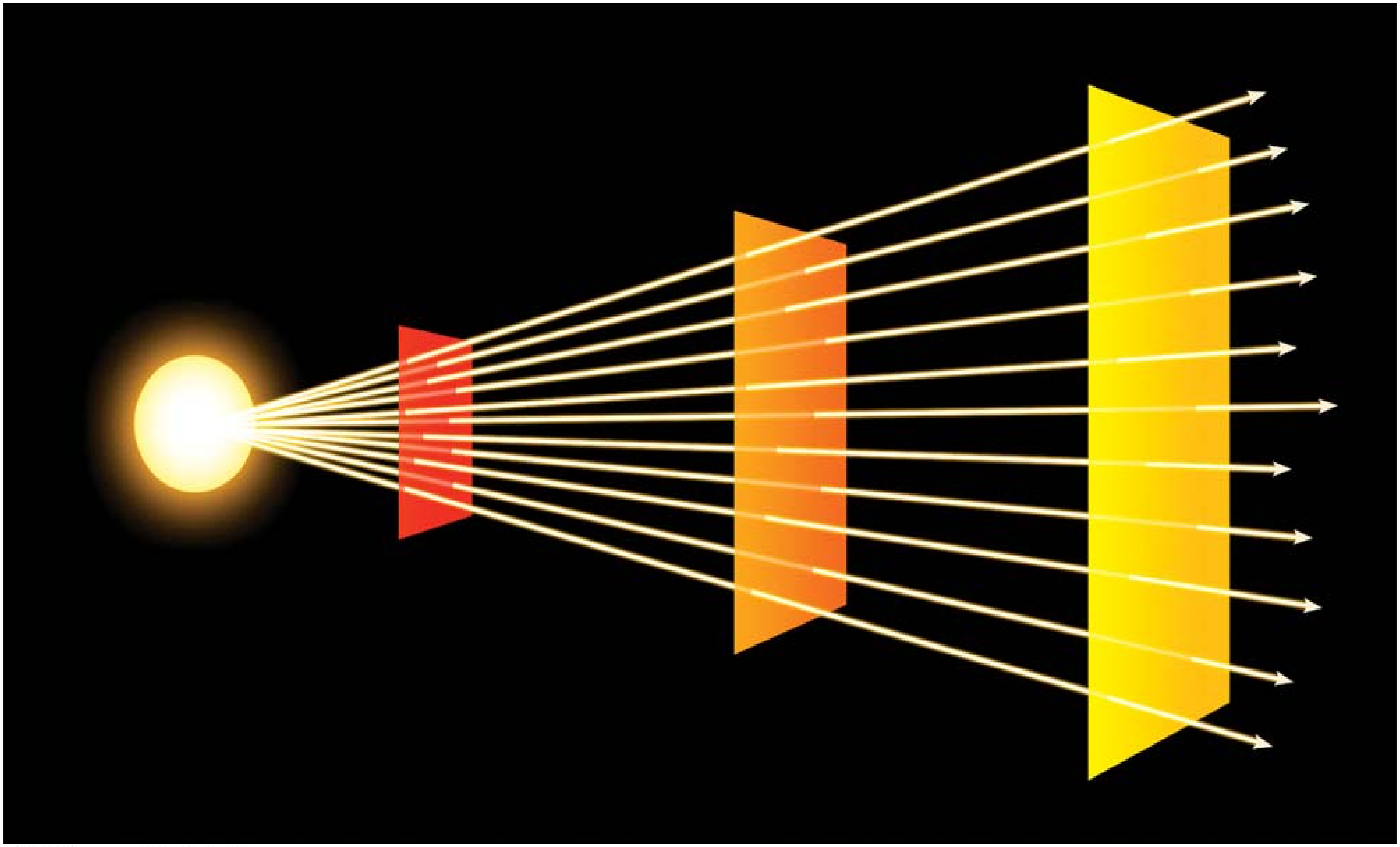
- Review of concepts from Lecture # 4
- Handouts-supplements
- Examples how to compute:
  - Earth-sun distance
  - Solar time
  - Declination
  - Solar zenith angle

- Reflectance/albedo

Earth-sun distance factor



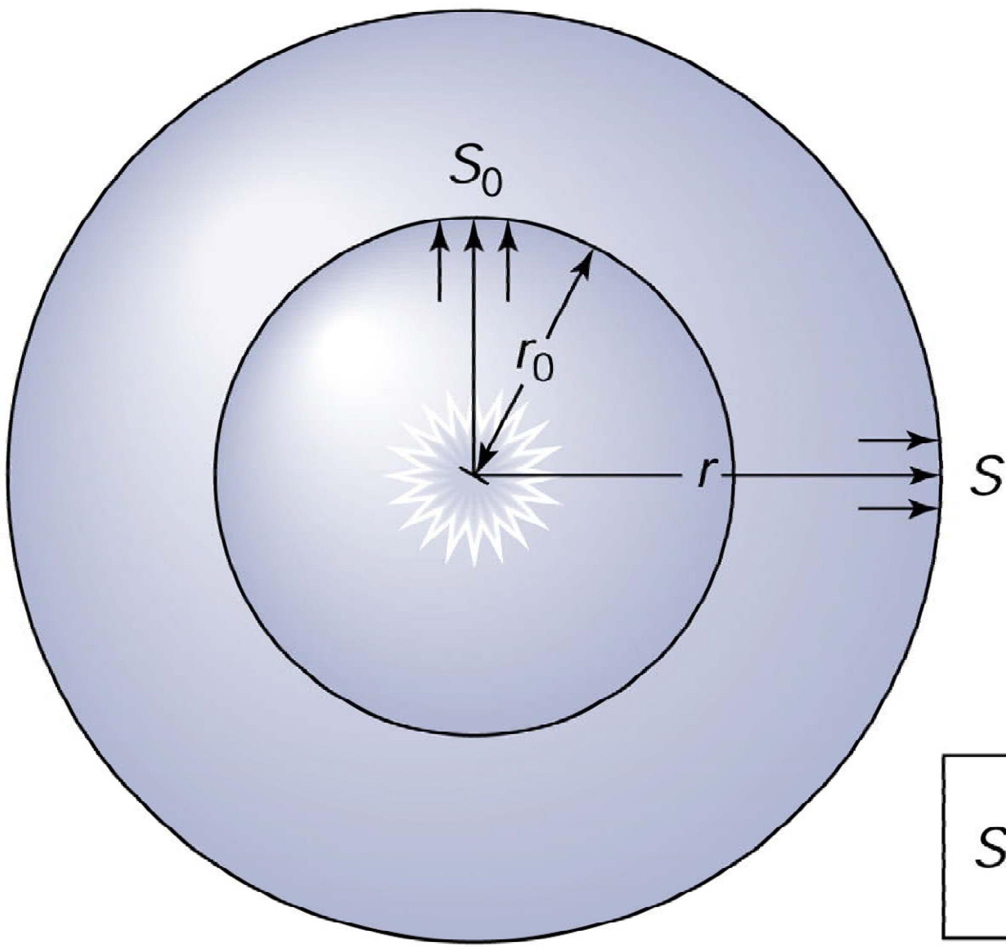
Laws that apply to solar radiation:
The Inverse Square Law



As you go away from the sun, the same energy is distributed over a larger area resulting in less energy per unit area.

Basic laws affecting the amount of radiation received from the sun:

The Inverse Square Law



$$S = S_0\left(\frac{r_0}{r}\right)^2$$

Sun – Earth Distance (r)

- Earth revolves around sun in elliptical orbit with the sun in one of the foci. Amount of solar radiation reaching the earth is inversely proportional to the square of the distance from the sun.
- The mean sun-earth distance r_0 is called one astronomical unit:
- 1 AU=1.496x10^8 km
- Formula for the reciprocal of the square of the radius vector of the earth-the eccentricity correction factor of the earth orbit, E_0 is:

A simple way to determine the Earth – Sun distance Factor , ε_0

$$\varepsilon_0 = (r_0/r)^2 = 1.000110+0.034221\cos\Gamma+0.001280\sin\Gamma+0.000719\cos2\Gamma+0.000077\sin2\Gamma$$

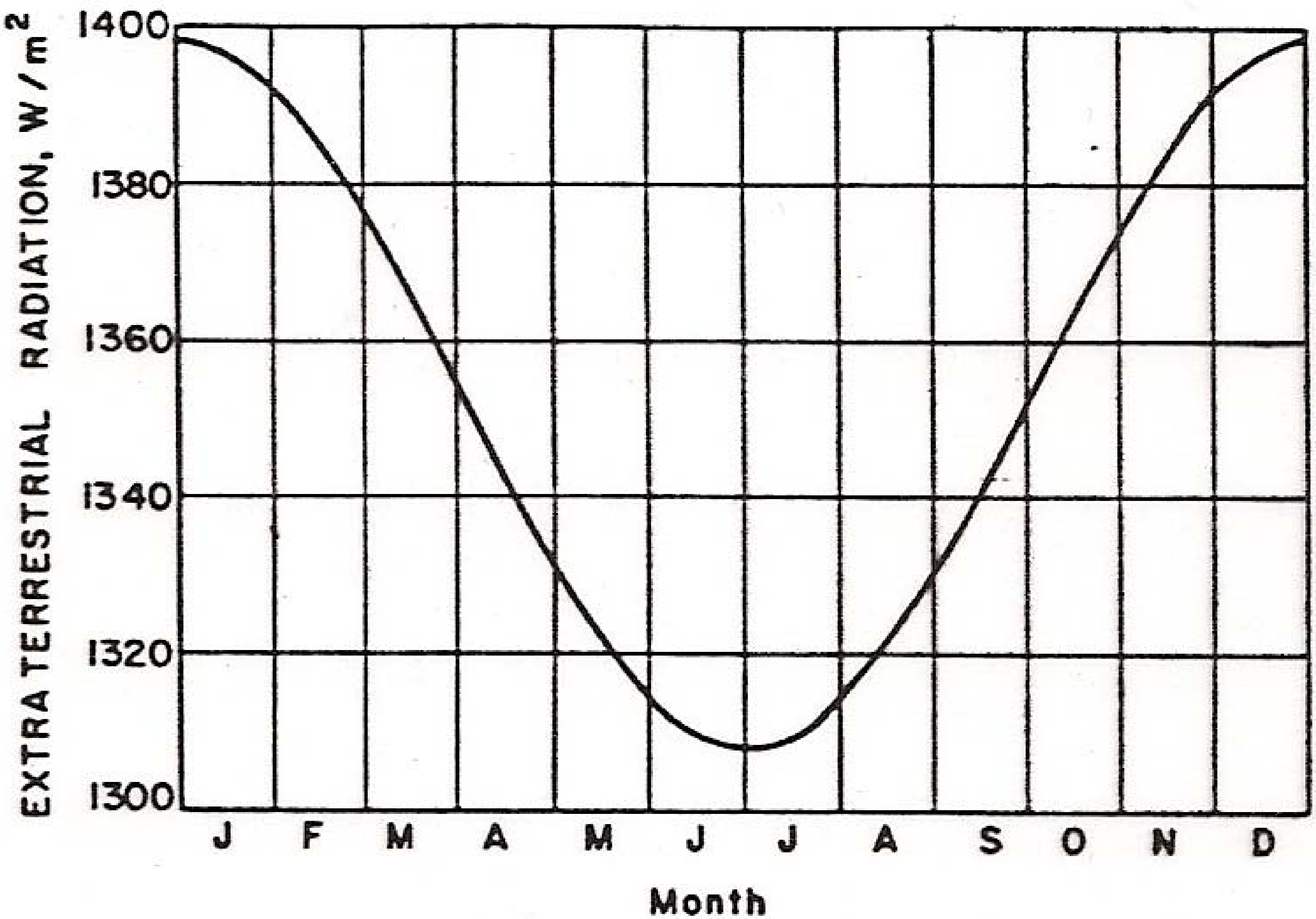
Here Γ is in radians and known as the day angle and equals:

$$\Gamma = 2\pi\,(d_n-1)/365$$

d_n is the day number of the year ranging from 1 on January 1 to 365 on December 31.

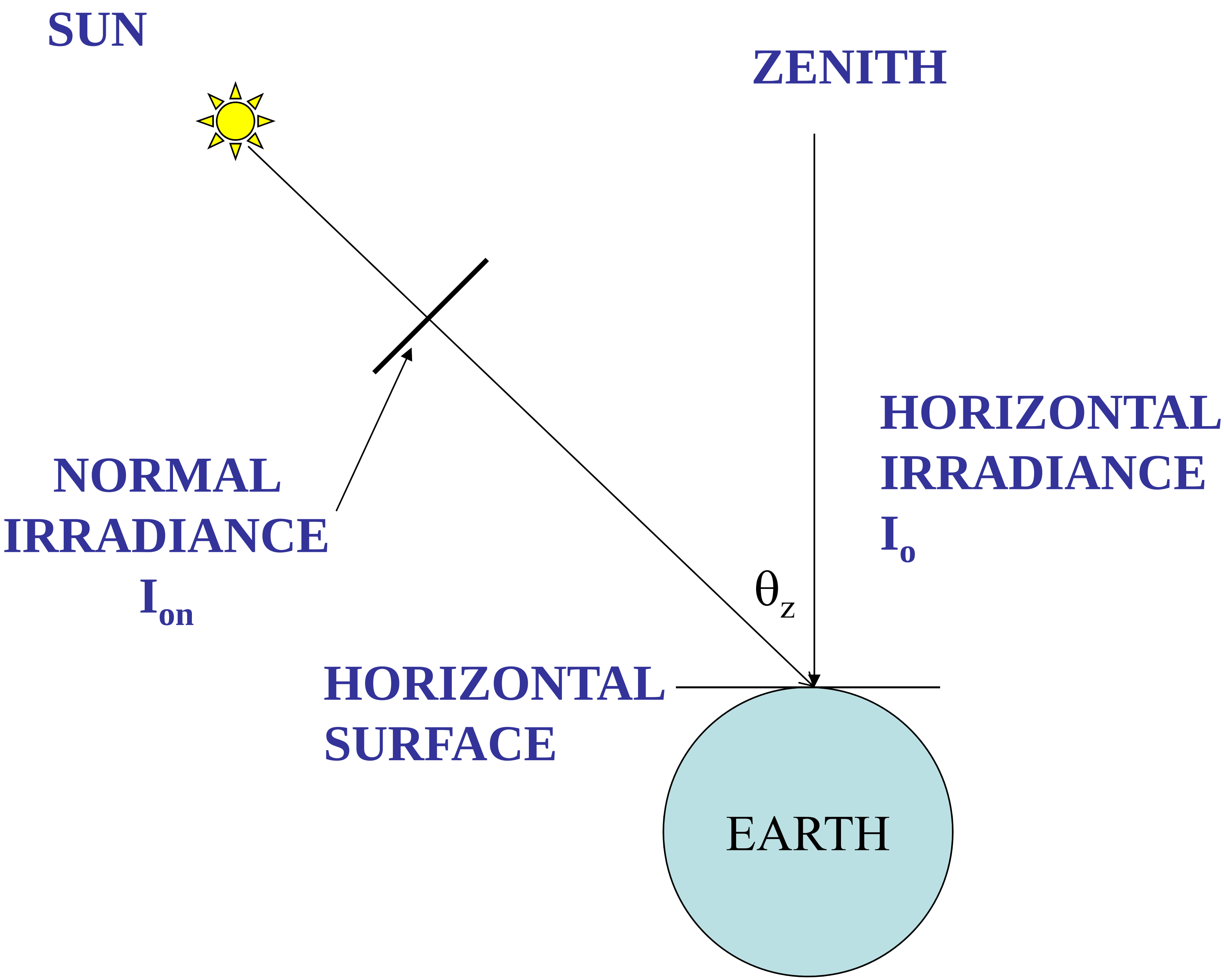
For engineering application:

$$\varepsilon_0 = (r_0/r)2 = 1+0.033\cos[(2\Pi d_n/365)]$$



Variation in earth-sun distance leads to variation in extraterrestrial flux in the range of (+-) 3%.

How to determine sun elevation θ?



The solar zenith angle - θ_z

- Intuitively, this angle depends on where on earth we are (latitude), what is the time of the day (measured in hour angles) and the season of the year (declination). Namely, the following parameters:
- φ – latitude
- ω hour angle
- δ - declination

- Each will be discussed in detail.

How to compute the solar zenith angle

Based on spherical geometry, the following relationship between the relevant angles is derived:

- $\cos\theta_z = \sin(\delta)\sin(\varphi) + \cos(\delta)\cos(\varphi)\cos(\omega)$
- φ – latitude
- δ - declination
- ω hour angle
- Hour angle ω is the distance in angle units from the solar noon (one hour is 15 deg). So first we need to derive the solar time for the location under consideration.
- Solar time = standard time + E + 4 ($L_{st} - L_{loc}$)

- E - equation in time in minutes
- L_{st} - standard meridian for local time zone
- L_{loc} - longitude of location in degrees west

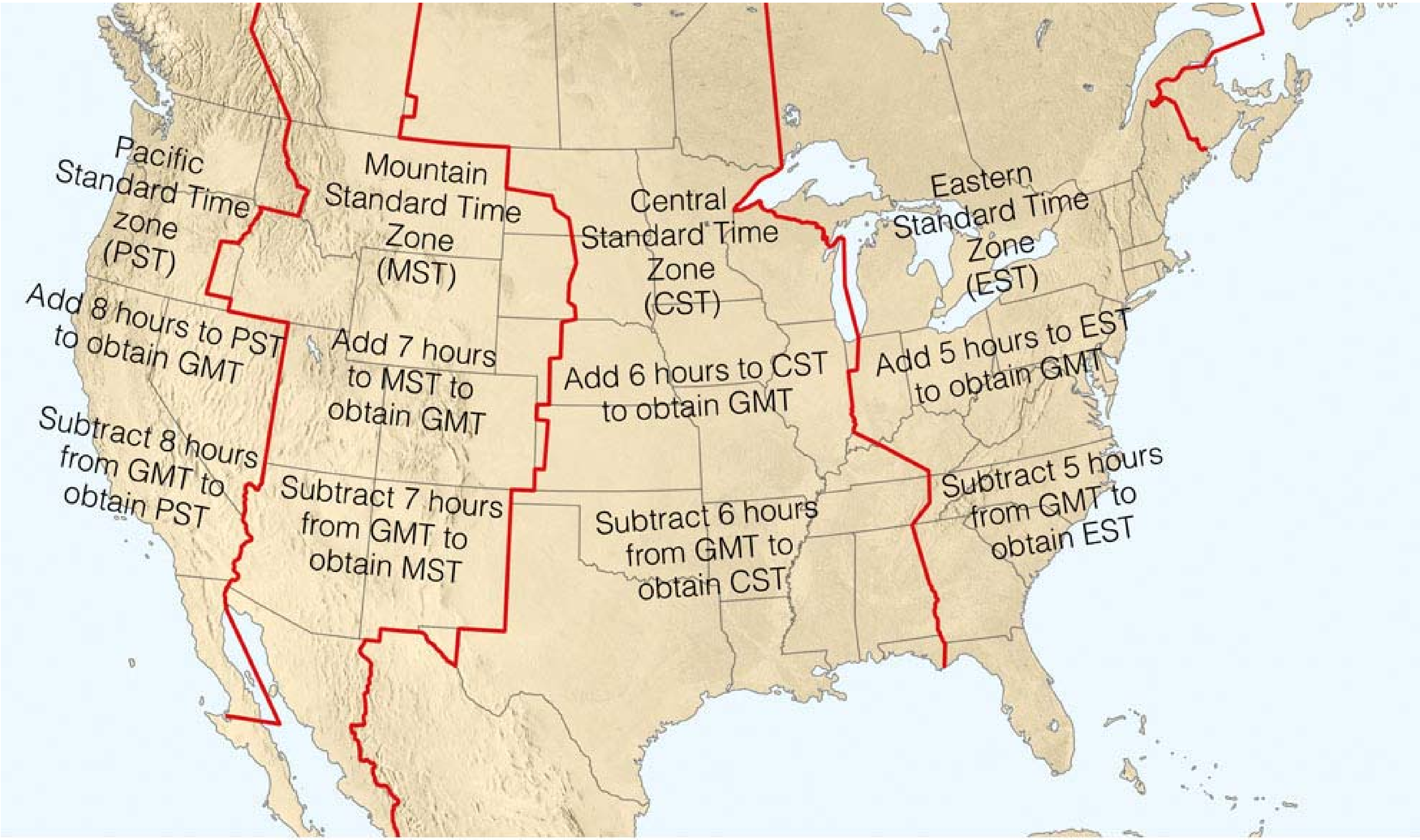
Solar Time

Time based on the *apparent angular motion of the sun across the sky*, with solar noon the time the sun crosses the meridian of the observer.

It is necessary to *convert* standard time to solar time by applying two corrections:

First-a constant correction for the difference in longitude between the observer's meridian and the meridian on which the local standard time is based. The sun takes 4 minutes to transverse 1^0 longitude.

Time zones



A **time zone** is a region on Earth that has a uniform standard time for legal, commercial, and social purposes.

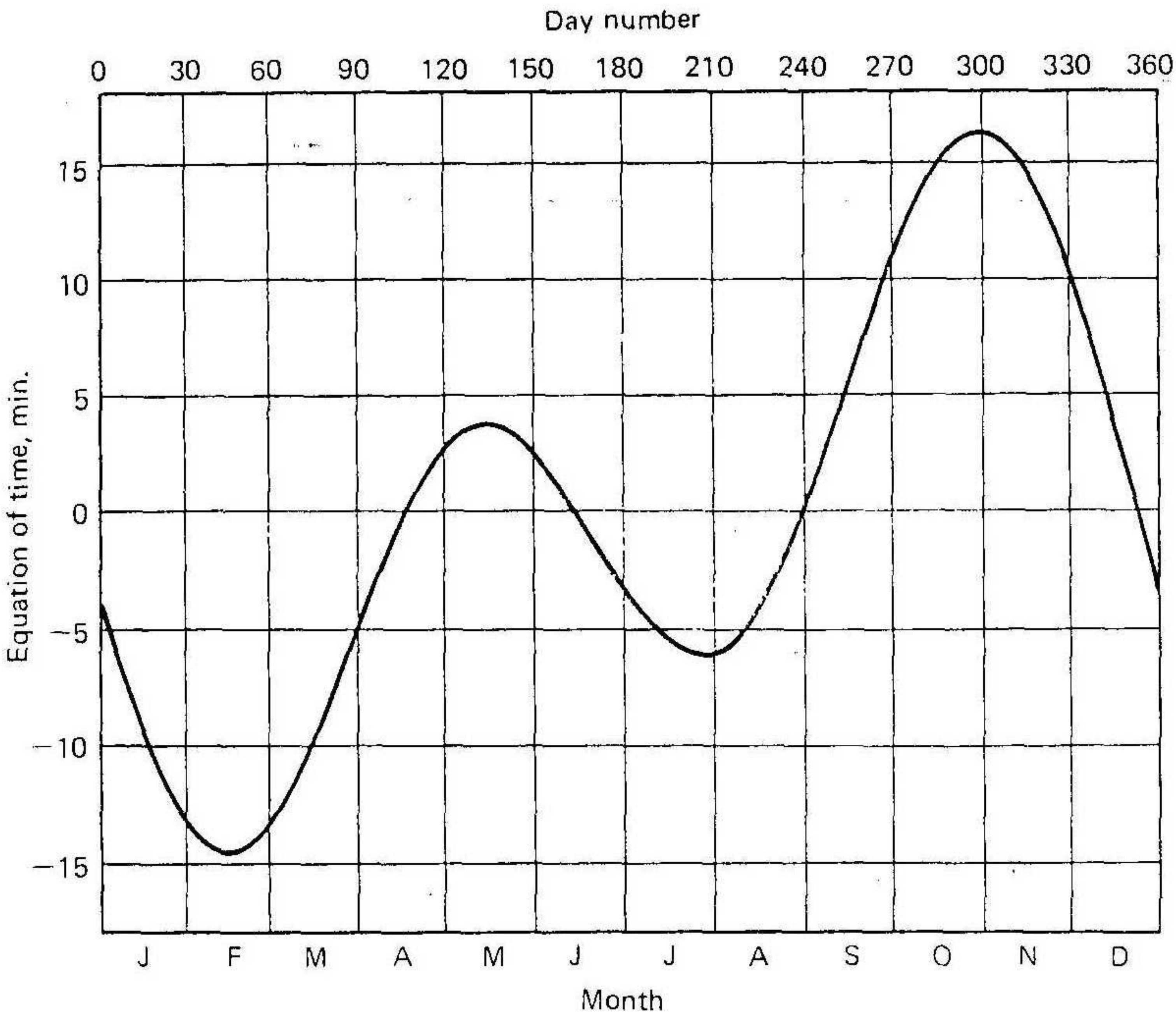
- **Second** - *equation of time.*

- Takes into account the perturbation in the earth rate of rotation which affect the time the sun crosses the observer's meridian.
- Solar time is:

- Solar time = standard time + $4(L_{st} - L_{loc}) + E$
- L_{st} is the standard meridian for local time zone
- L_{loc} is the longitude of the location in degrees west

- $E = (0.000075+0.001868\cos\Gamma-0.032077\sin\Gamma-0.014615\cos2\Gamma-0.04089\sin2\Gamma)\ (229.18)$

- The number 229.18 converts radians into minutes. $\Gamma = 2\pi(d_n-1)/365$



Equation of time, E, in minutes, as a function of time of year.

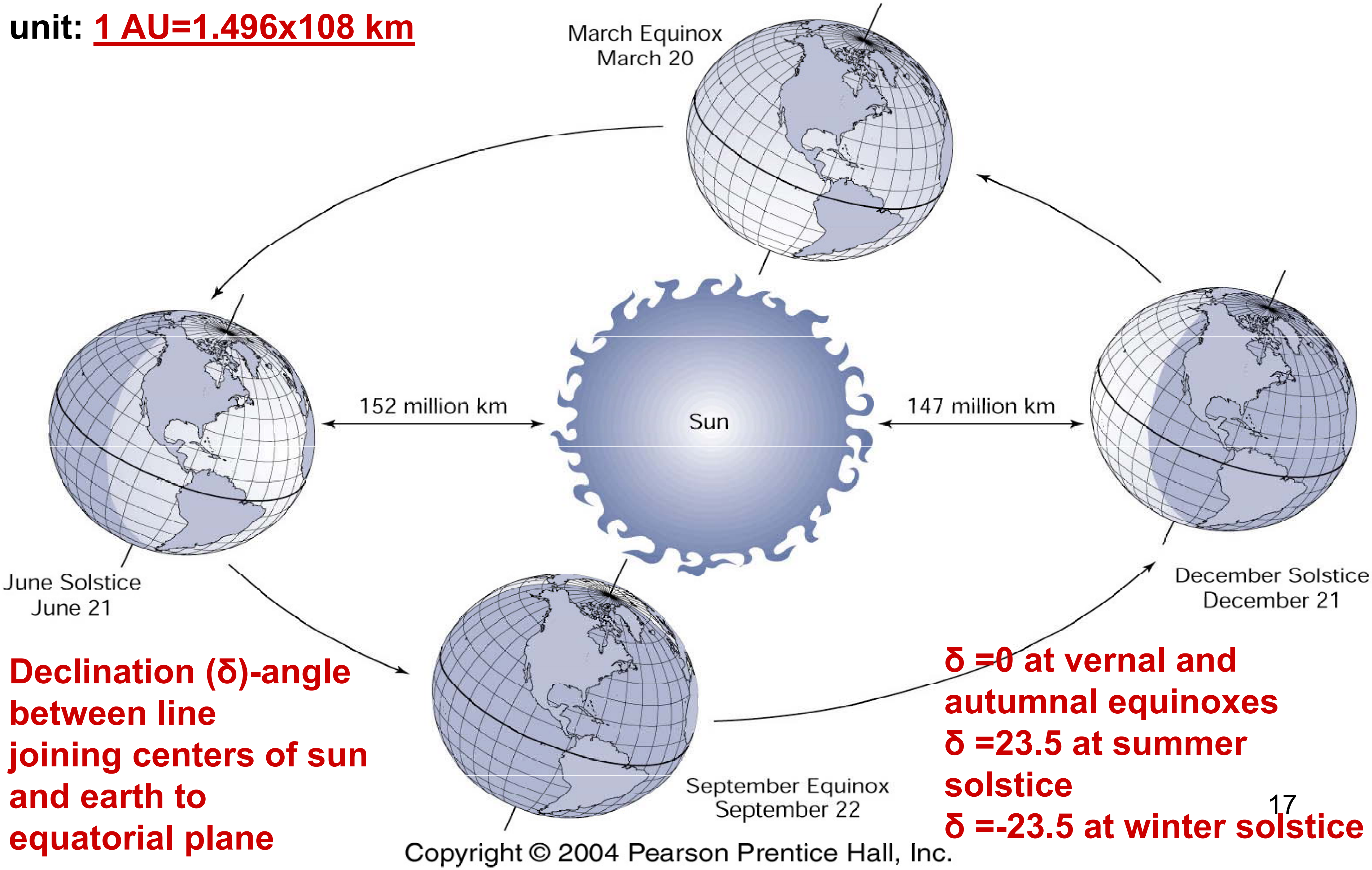
Solar declination

- The angle between a line joining the centers of the sun and the earth to the equatorial plane changes every day - *the solar declination*. It is zero at vernal and autumnal equinoxes and has a value of 23.5^0 at summer solstice and – 23.5^0 in winter solstice.

- δ = (0.006918-0.399912cosΓ-
- 0.070257sinΓ-0.006758cos2Γ-
- 0.000907sin2Γ-0.002697cos3Γ-
- 0.00148sin3Γ)(180/π)

The Seasons

Mean sun-earth distance is one astronomical unit: 1 AU=1.496x108 km



Declination (δ)-angle between line joining centers of sun and earth to equatorial plane

δ =0 at vernal and autumnal equinoxes
δ =23.5 at summer solstice
δ =-23.5 at winter solstice

By now, you have all the tools to compute the solar zenith angle

- $\cos\theta_z = \sin(\delta)\sin(\varphi) + \cos(\delta)\cos(\varphi)\cos(\omega)$
- φ – latitude
- δ - declination
- ω hour angle

- Solar time = standard time + E + 4 ($L_{st} - L_{loc}$)

- E - equation in time in minutes
- L_{st} - standard meridian for local time zone

- L_{loc} - longitude of location in degrees west

TABLE 2.4 Summary Solar Ephemeris[a]

	Declination		Equation of time			Declination		Equation of time	
Date	Deg	Min	Min	Sec	Date	Deg	Min	Min	Sec
Jan. 1	−23	4	−3	14	Feb. 1	−17	19	−13	34
5	22	42	5	6	5	16	10	14	2
9	22	13	6	50	9	14	55	14	17
13	21	37	8	27	13	13	37	14	20
17	20	54	9	54	17	12	15	14	10
21	20	5	11	10	21	10	50	13	50
25	19	9	12	14	25	9	23	13	19
29	18	8	13	5					
Mar. 1	−7	53	−12	38	Apr. 1	+4	14	−4	12
5	6	21	11	48	5	5	46	3	1
9	4	48	10	51	9	7	17	1	52
13	3	14	9	49	13	8	46	−0	47
17	1	39	8	42	17	10	12	+0	13
21	−0	5	7	32	21	11	35	1	6
25	+1	30	6	20	25	12	56	1	53
29	3	4	5	7	29	14	13	2	33
May 1	+14	50	+2	50	June 1	+21	57	+2	27
5	16	2	3	17	5	22	28	1	49
9	17	9	3	35	9	22	52	1	6
13	18	11	3	44	13	23	10	+0	18
17	19	9	3	44	17	23	22	−0	33
21	20	2	3	34	21	23	27	1	25
25	20	49	3	16	25	23	25	2	17
29	21	30	2	51	29	23	17	3	7
July 1	+23	10	−3	31	Aug. 1	+18	14	−6	17
5	22	52	4	16	5	17	12	5	59
9	22	28	4	56	9	16	6	5	33
13	21	57	5	30	13	14	55	4	57
17	21	21	5	57	17	13	41	4	12
21	20	38	6	15	21	12	23	3	19
25	19	50	6	24	25	11	2	2	18
29	18	57	6	23	29	9	39	1	10
Sep. 1	+8	35	−0	15	Oct. 1	−2	53	+10	1
5	7	7	+1	2	5	4	26	11	17
9	5	37	2	22	9	5	58	12	27
13	4	6	3	45	13	7	29	13	30
17	2	34	5	10	17	8	58	14	25
21	+1	1	6	35	21	10	25	15	10
25	−0	32	8	0	25	11	50	15	46
29	2	6	9	22	29	13	12	16	10
Nov. 1	−14	11	+16	21	Dec. 1	−21	41	+11	16
5	15	27	16	23	5	22	16	9	43
9	16	38	16	12	9	22	45	8	1
13	17	45	15	47	13	23	6	6	12
17	18	48	15	10	17	23	20	4	17
21	19	45	14	18	21	23	26	2	19
25	20	36	13	15	25	23	25	+0	20
29	21	21	11	59	29	23	17	−1	39

[a] Since each year is 365.25 days long, the precise value of declination varies from year to year. *The American Ephemeris and Nautical Almanac* published each year by the U.S. Government Printing Office contains precise values for each day of each year.

4.2 Extraterrestrial Irradiation on a Horizontal Surface

The expressions for radiation on horizontal surfaces will be formulated for different time periods: an hour, a day, a month, and so forth.

A. *Hourly Radiation on a Horizontal Surface*

On a given day, let $\dot{I}_{0n}$ be the extraterrestrial irradiance (*rate* of energy) on a surface normal to the rays from the sun, where

$$\dot{I}_{0n} = \dot{I}_{SC}(r_0/r)^2 = \dot{I}_{SC}E_0. \tag{4.2.1}$$

It is obvious from Fig. 4.2.1 that the irradiance on a horizontal surface can be written

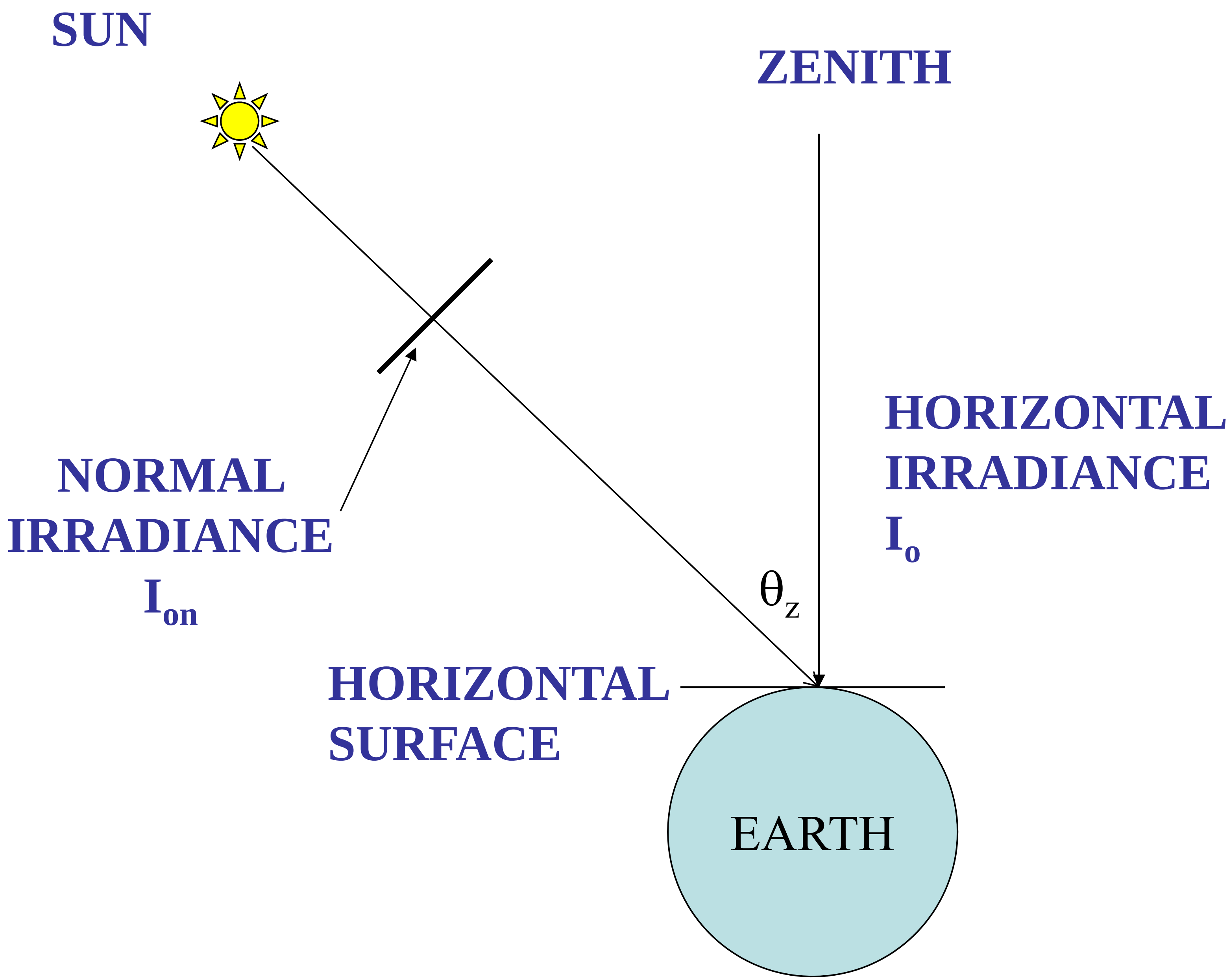
$$\dot{I}_0 = \dot{I}_{0n}\cos\theta_z, \tag{4.2.2}$$

where $\cos\theta_z$ is given by Eq. (1.5.1) or

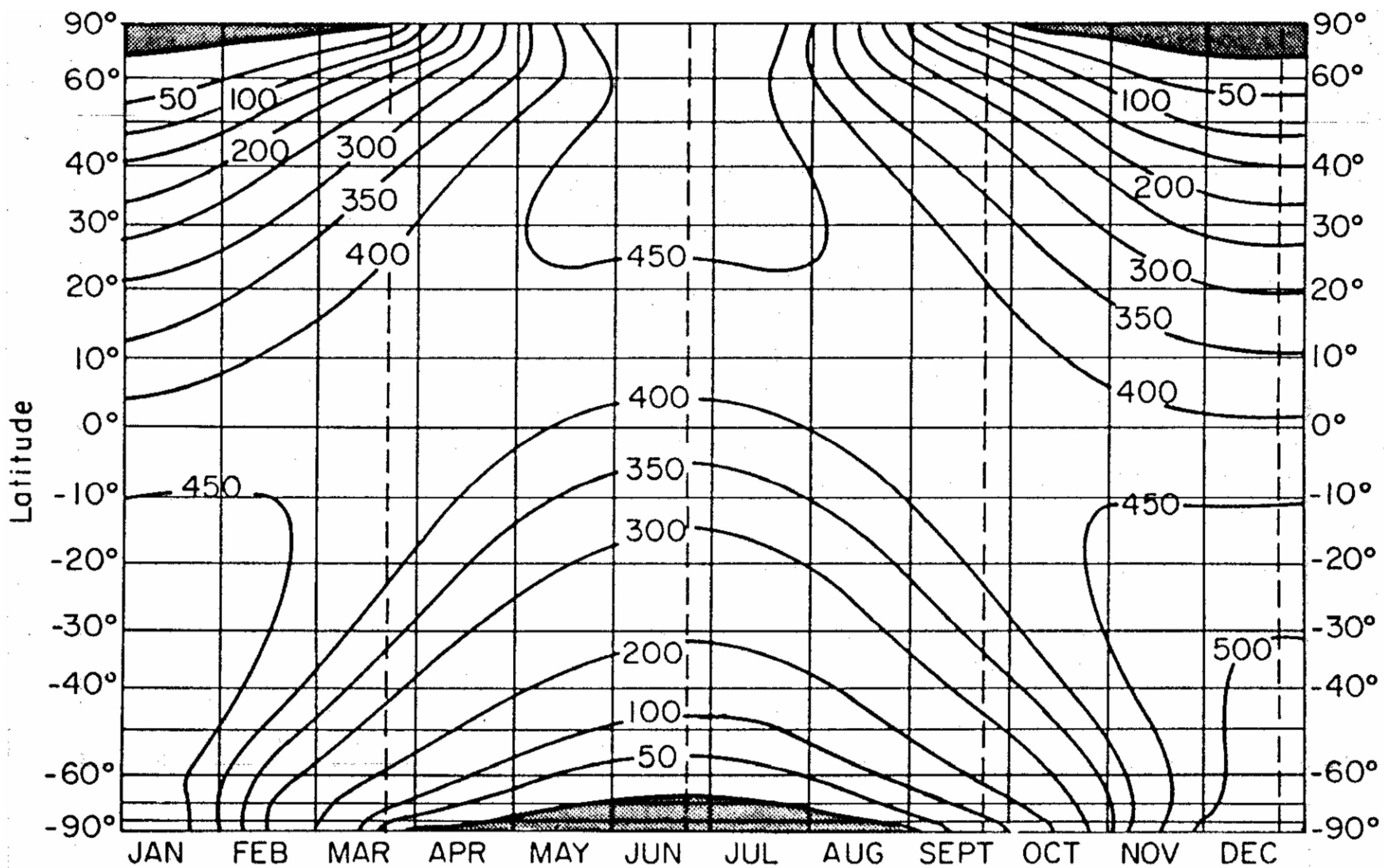
$$\dot{I}_0 = \dot{I}_{SC}E_0(\sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega). \tag{4.2.3}$$

The units of Eqs. (4.2.1)–(4.2.3) are W m^{-2}.

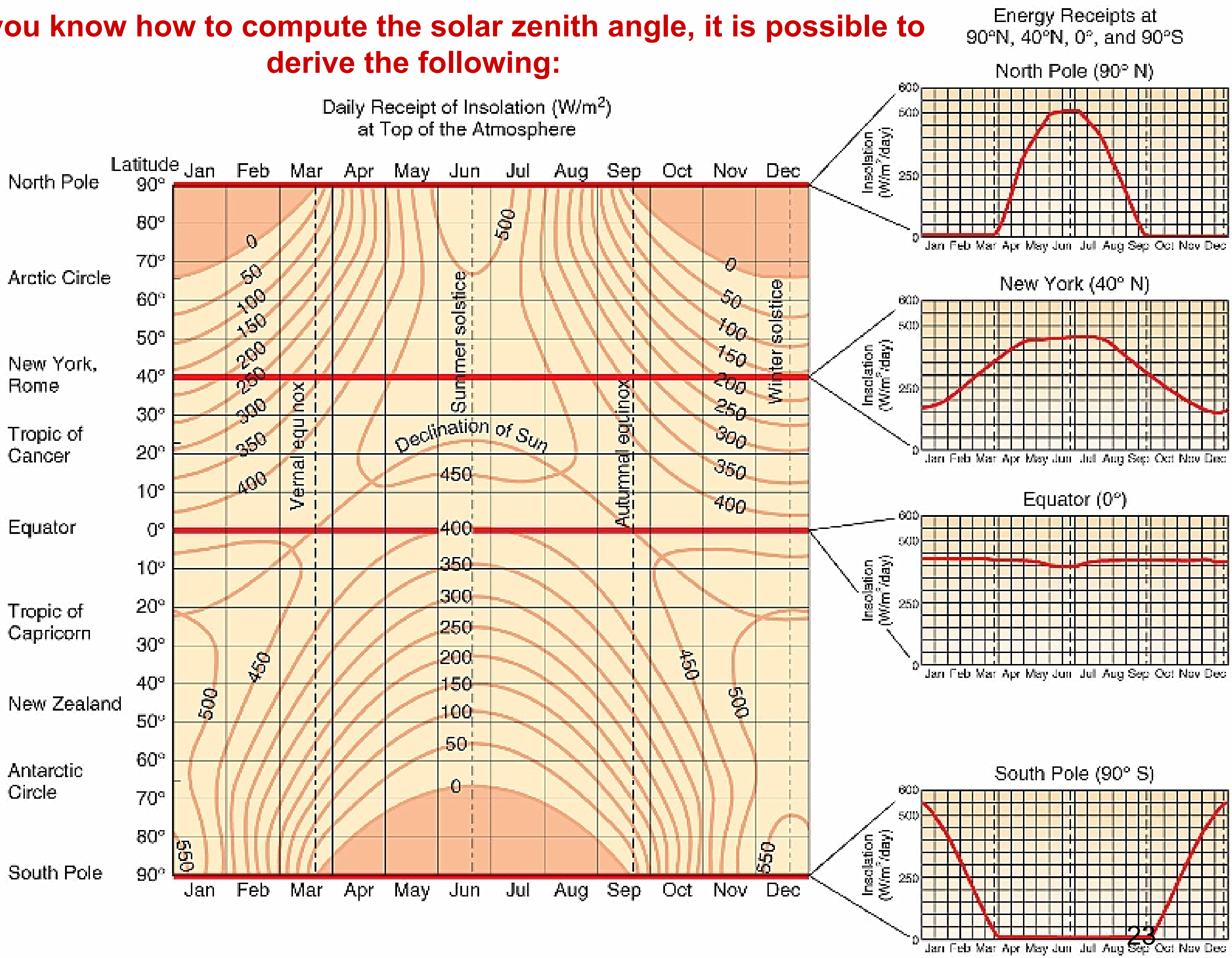
How to determine sun elevation θ?



Daily solar insolation in W/m² incident on a horizontal surface at the top of the atmosphere as a function of latitude and date (adapted from Milankovitch, 1930).



Once you know how to compute the solar zenith angle, it is possible to derive the following:



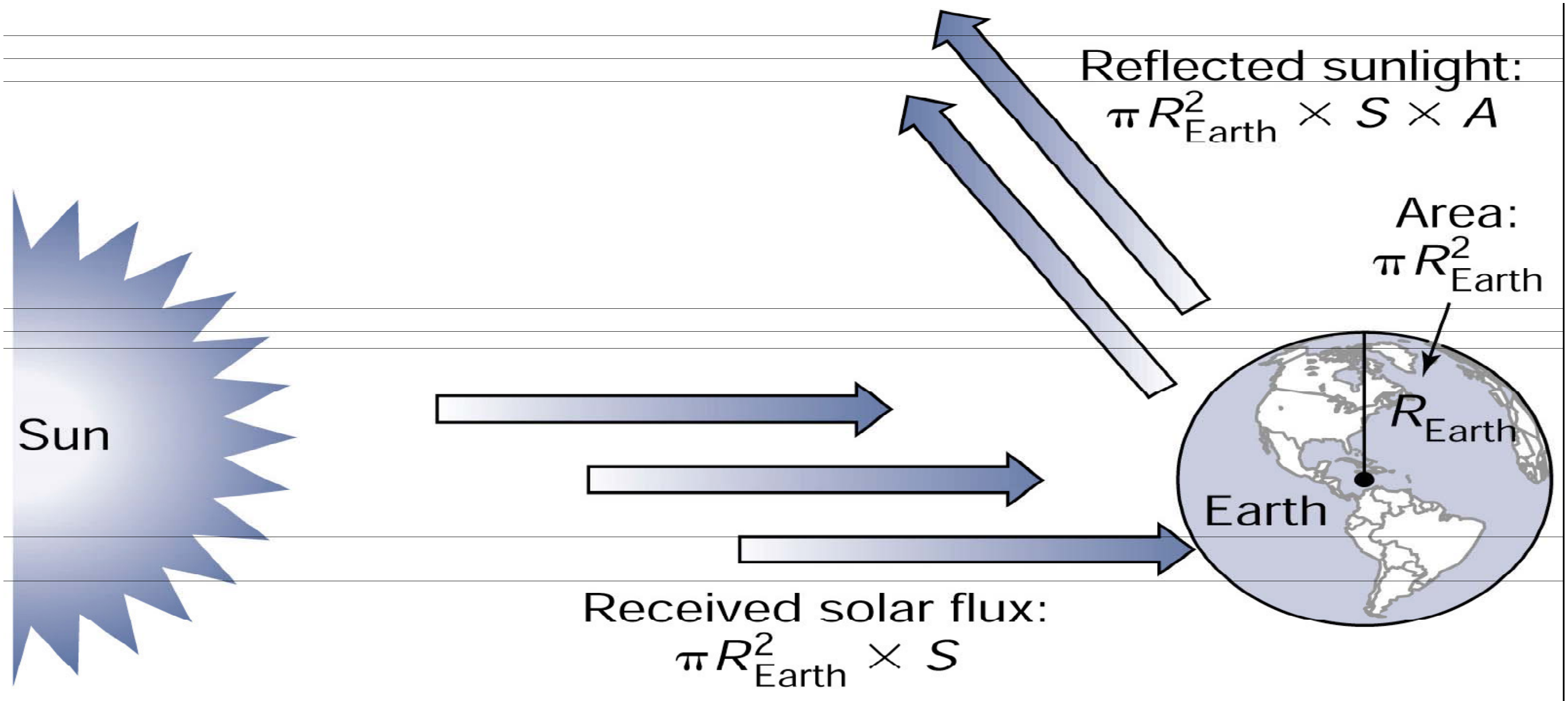
Once you know how to derive the solar zenith angle, you can estimate the length of the day. How?

Table 2.3 Daylength Times (Sunrise and Sunset) at Selected Latitudes (Northern Hemisphere)

	Winter Solstice (December Solstice) December 21–22			Vernal Equinox (March Equinox) March 20–21			Summer Solstice (June Solstice) June 20–21			Autumnal Equinox (September Equinox) September 22–23		
Latitude	A.M.	P.M.	Daylength	A.M.	P.M.	Daylength	A.M.	P.M.	Daylength	A.M.	P.M.	Daylength
0°	6:00	6:00	12:00	6:00	6:00	12:00	6:00	6:00	12:00	6:00	6:00	12:00
30°	6:58	5:02	10:04	6:00	6:00	12:00	5:02	6:58	13:56	6:00	6:00	12:00
40°	7:30	4:30	9:00	6:00	6:00	12:00	4:30	7:30	15:00	6:00	6:00	12:00
50°	8:05	3:55	7:50	6:00	6:00	12:00	3:55	8:05	16:10	6:00	6:00	12:00
60°	9:15	2:45	5:30	6:00	6:00	12:00	2:45	9:15	18:30	6:00	6:00	12:00
90°	No sunlight			Rising Sun			Continuous sunlight			Setting Sun		

Note: All times are standard and do not consider the local option of daylight saving time.

The Energy Source for Weather and Climate is Solar Radiation from the Sun



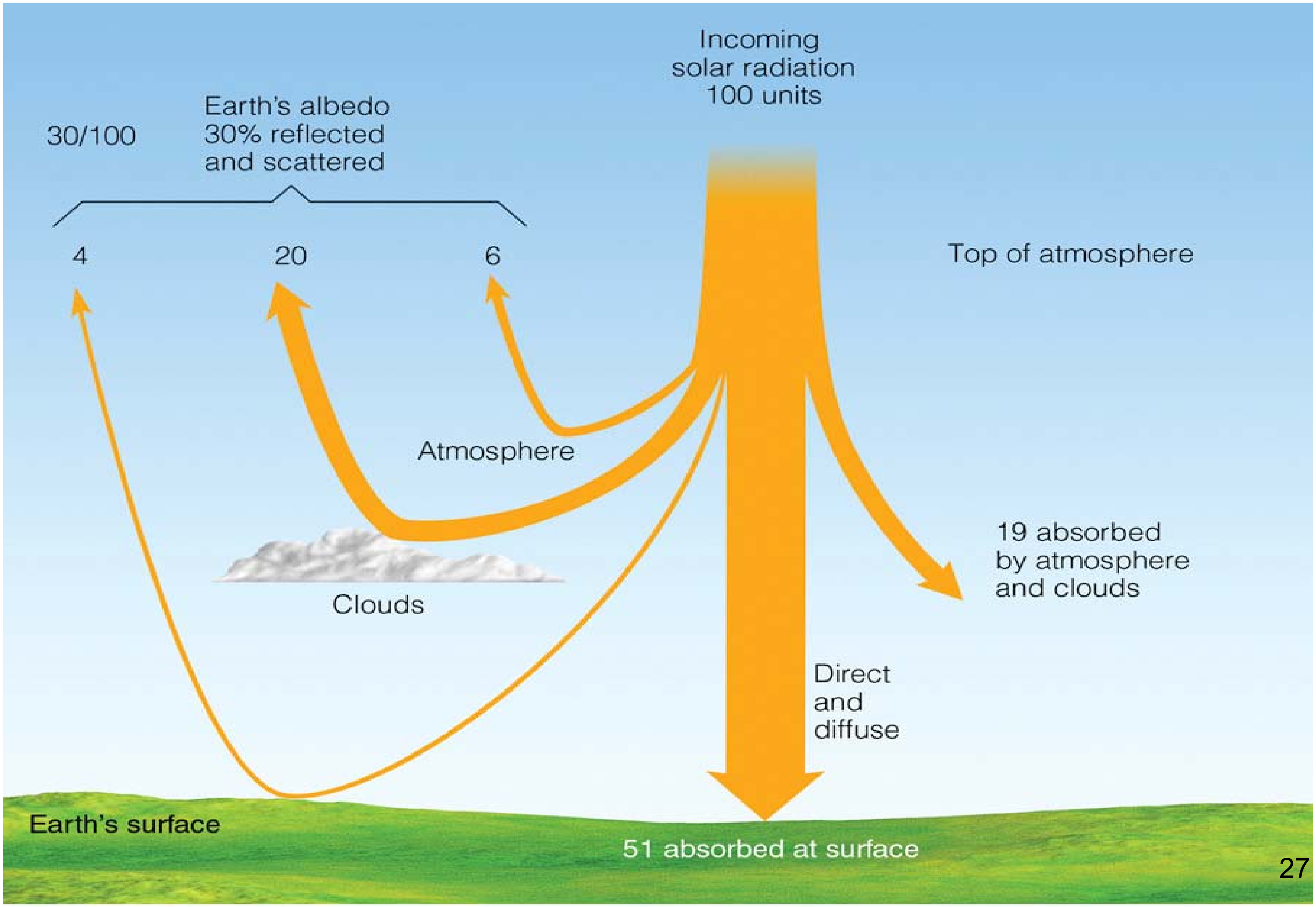
Of interest: what fraction of received goes back

The ratio between the *reflected* part and the *incoming* part is called albedo (A)

Typical Albedo of Various Surfaces

SURFACE	ALBEDO (PERCENT)
Fresh snow	75 to 95
Clouds (thick)	60 to 90
Clouds (thin)	30 to 50
Venus	78
Ice	30 to 40
Sand	15 to 45
Earth and atmosphere	30
Mars	17
Grassy field	10 to 30
Dry, plowed field	5 to 20
Water	10*
Forest	3 to 10
Moon	7

Shortwave (solar) Radiation Budget



Earth's albedo that includes clouds can be estimated from satellites

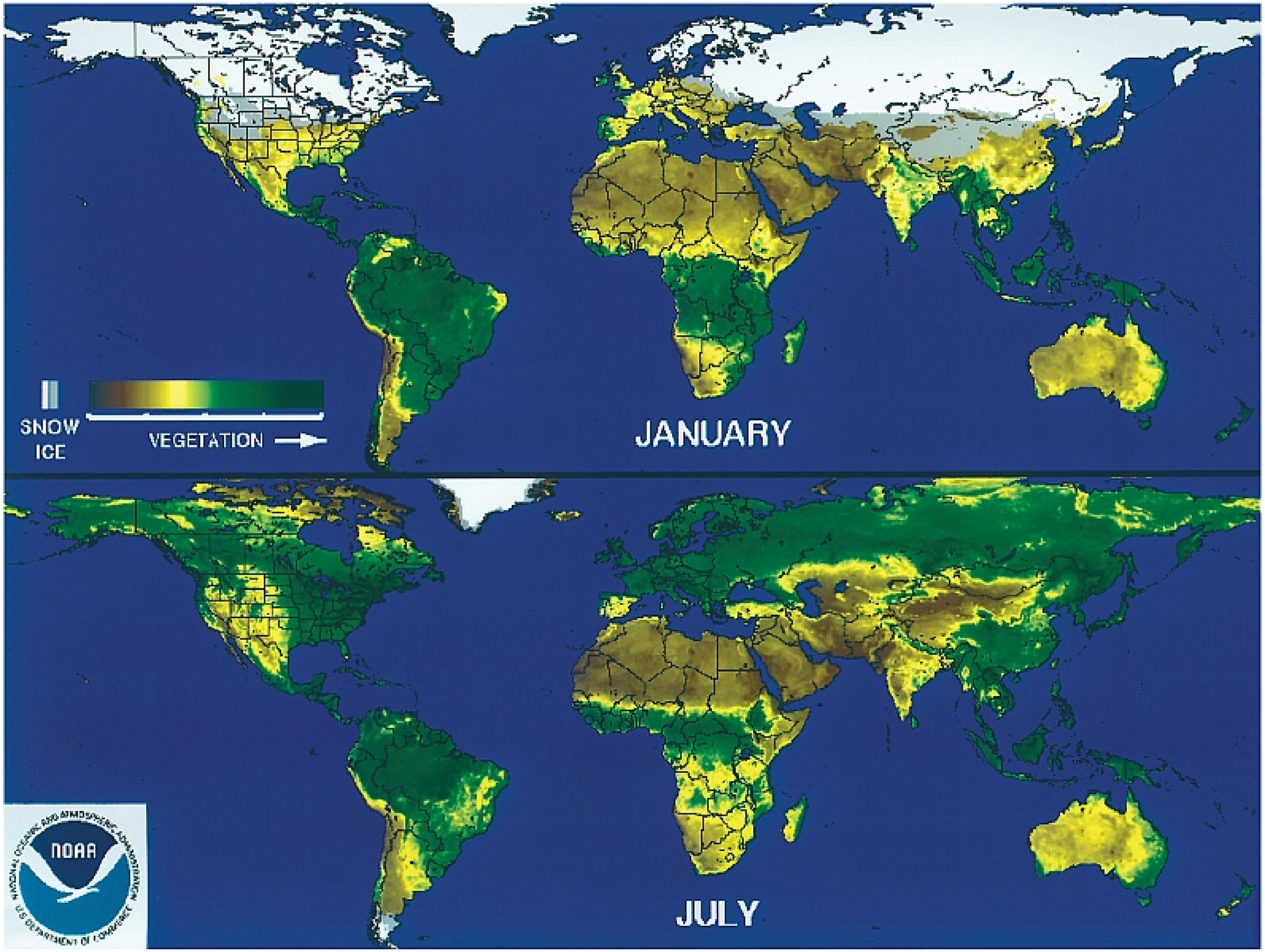
The various elements that affect the Earth's albedo cover a large range of values:

Water: 10%

Snow: 80-90%

Desert sand: 40%

Earth average: 31%



Differences in surface albedo in summer and winter

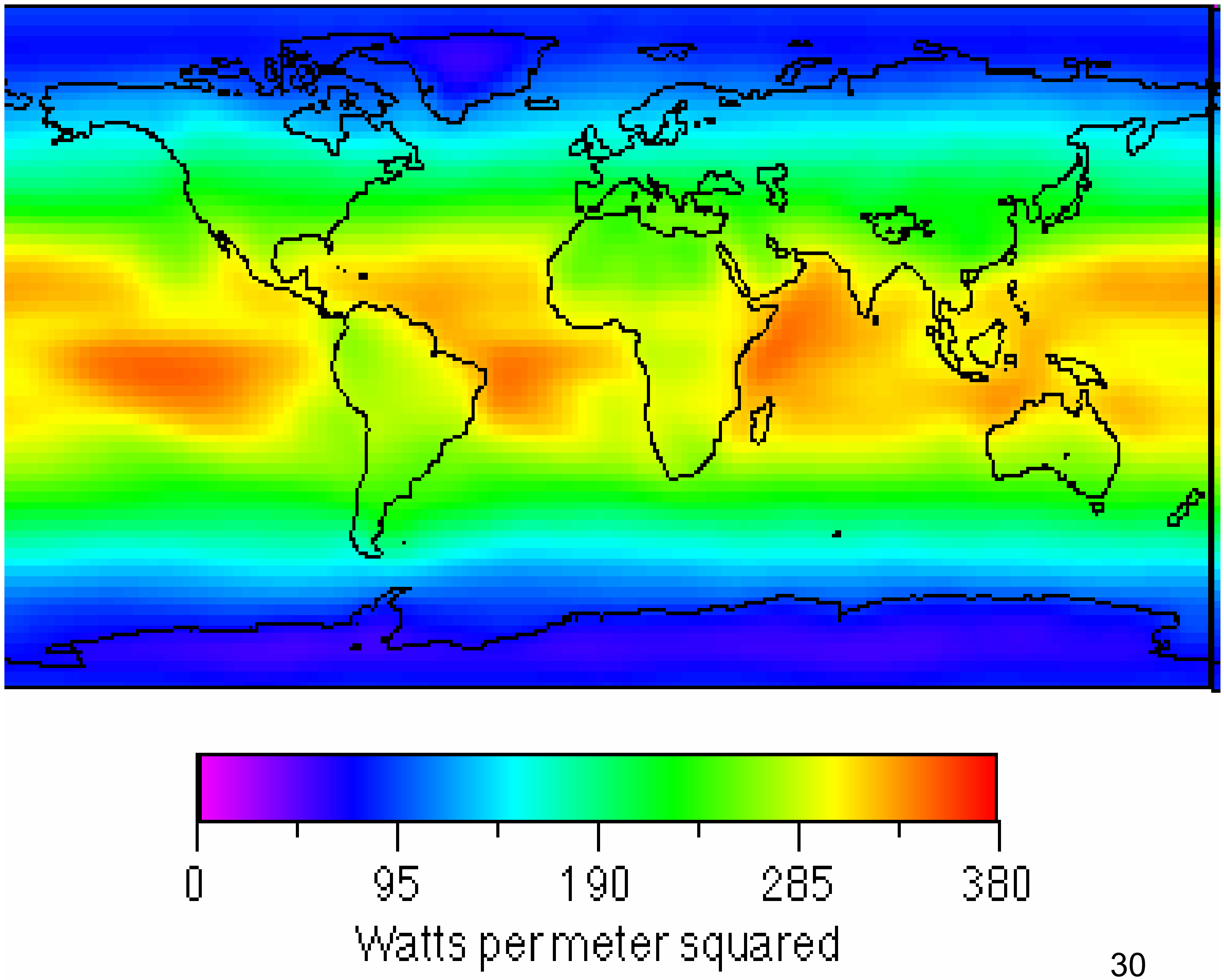
Annual Average Surface Downward Shortwave

Top of atm: ~1400W/m^2

Albedo effect: 700W/m ^{2}

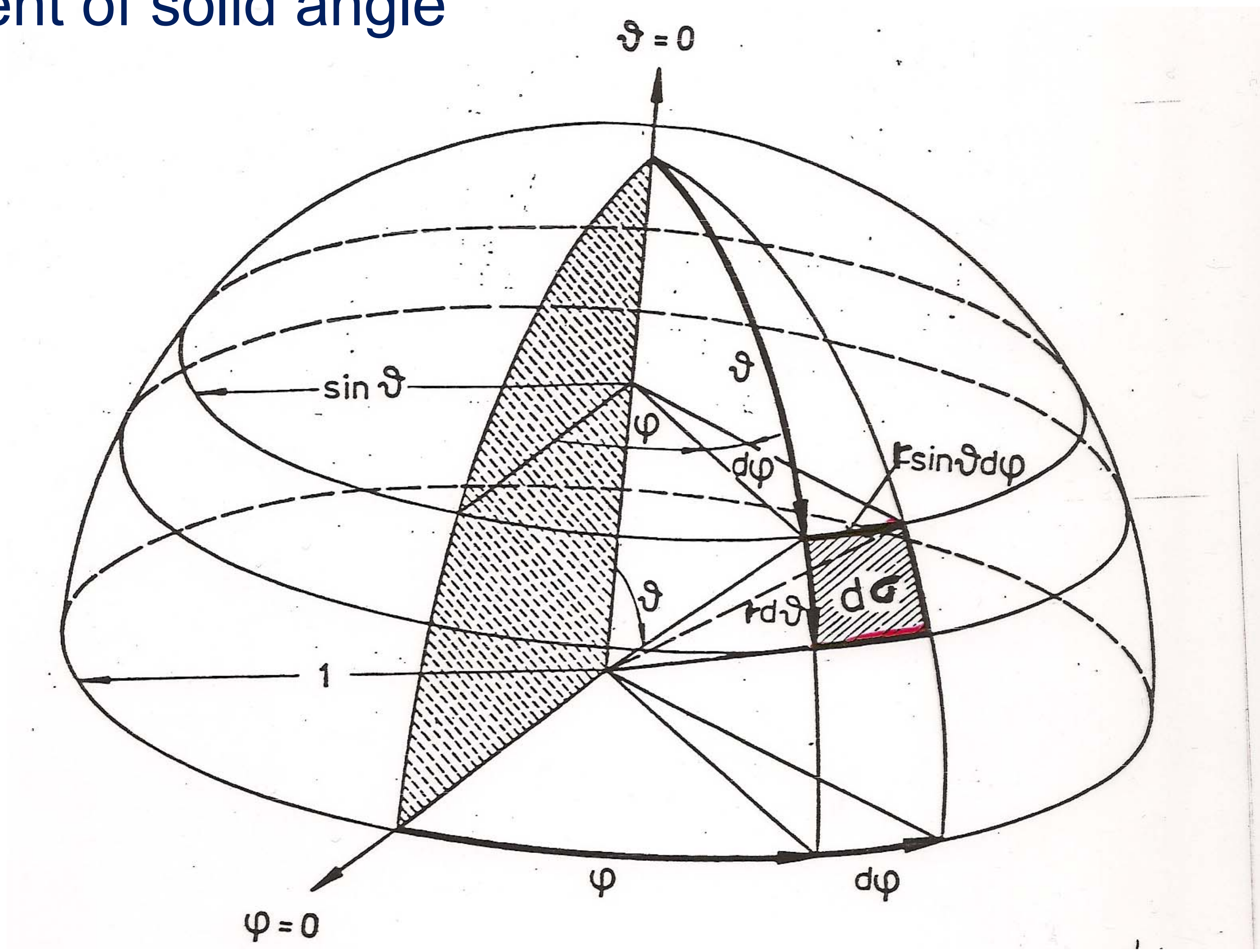
Due to day/night cycle: 350W/m^2

average per day



Review of concept for understanding radiometric quantities (Table to follow)

Element of solid angle



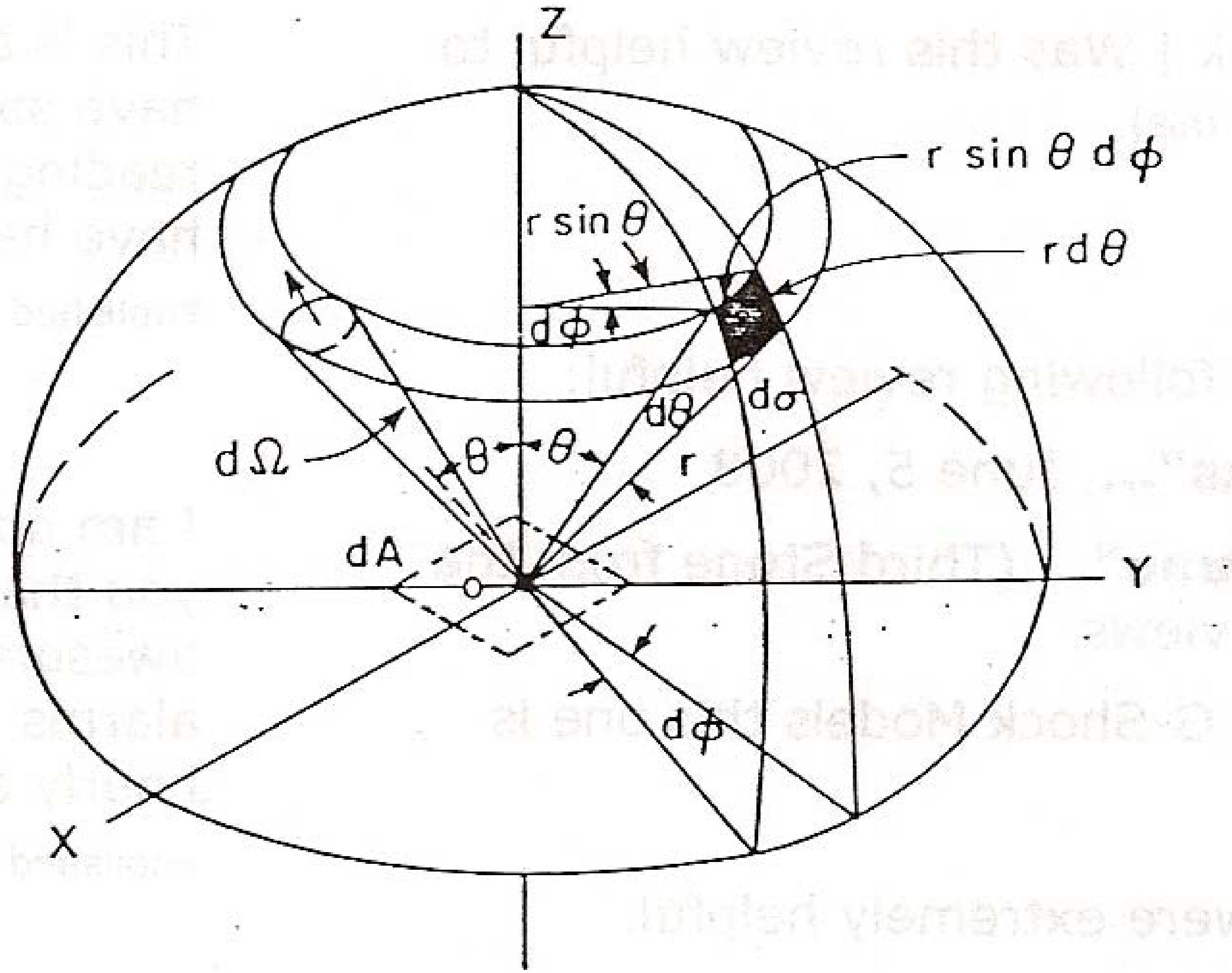


Illustration of a solid angle in polar coordinates and a pencil of radiation through an element of area dA in directions confined to an element of solid angle dΩ

Solid angle is defined as the ratio of the area σ of a spherical surface intercepted by the cone to the square of the radius r, namely:

$$\Omega = \sigma / r^2$$

Units of solid angle are expressed in terms of the steradian (sr)
For a sphere of surface area $4\pi r^2$, its solid angle is 4π sr.
A differential element of solid angle:

$$d\sigma = (r\, d\theta)(r \sin\theta\, d\phi).$$

Hence, the differential solid angle is

$$d\Omega = d\sigma / r^2 = \sin\theta\, d\theta\, d\phi,$$

where θ and ϕ denote the zenithal and azimuthal angles, polar coordinates.

Table 1: Radiometric quantities (described in Section 3). Symbols in brackets are proposed for alternative use.

NAMES	SYMBOL	UNIT	RELATION	REMARKS	CIE-no.
radiant energy	Q, (W)	J = W s			45-05-130
radiant flux	Φ, (P)	W	$\Phi = \frac{dQ}{dt}$	power	45-05-135
radiant flux density	(M), (E)	$W\ m^{-2}$	$\frac{d\Phi}{dA} = \frac{d^2Q}{dA\ dt}$	Radiant flux of any origin <u>crossing</u> an area element	45-05-155
radiant exitance*	M	$W\ m^{-2}$	$M = \frac{d\Phi}{dA}$	Radiant flux of any origin <u>emerging</u> from an area element	45-05-170
irradiance	E	$W\ m^{-2}$	$E = \frac{d\Phi}{dA}$	Radiant flux of any origin <u>incident</u> onto an area element	45-05-160
radiance	L	$W\ m^{-2} sr^{-1}$	$L = \frac{d^2\Phi}{d\Omega\ dA\ \cos\vartheta}$	The radiance is a conservative quantity in an optical system	45-05-150
radiant exposure	H	$J\ m^{-2}$ (per exposure time)	$H = \frac{dQ}{dA} = \int_{t_1}^{t_2} E\ dt$ t_1, t_2: time	May be used for daily sums of global radiation, etc.	45-05-165
radiant intensity	I	$W\ sr^{-1}$	$I = \frac{d\Phi}{d\Omega}$	May be used only for radiation outgoing from "point sources"	45-05-145

*The name radiant exitance has been proposed in CIE (1970) to avoid confusion with the name emittance which has previously been used for this quantity (see ...